Pumping Water to Compete in Electricity Markets

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Abstract

The pump storage technique allows to use cheap thermal electricity at periods of low demand to restore water resources that can be used to generate electricity at periods of peak demand. When the thermal plant and the hydro plant are managed by the same operator, the two plants are used efficiently. It means in particular that there is no sale of electricity by the hydroproducer during the period where it is buying electricity for pumping. This does not remain necessarily true when the two plants are operated by two independent private entities. The paper analyses the optimal dispatch and the profit-maximizing dispatch of the thermal and hydro-generation units when water can be pumped up into reservoirs. We then switch to the case where the two plants are operated by separate owners. We analyse the Cournot equilibrium of the game where the hydro unit is the client of the thermal unit at off-peak period, and compete against it at peak period. We show that in some cases the hydro unit can be better off by selling electricity at off-peak periods, because it allows to buy cheaper electricity for its pumping activity.

Keywords: water resource; pumping; hydroelectricity; Cournot competition

JEL classification: L12, L13, Q25, Q42
1 Introduction

Electricity can be generated by means of very heterogeneous technologies in nuclear plants, gas, coal or fuel-burning installations, windmills, hydroelectric plants, etc. These technologies are both complements and substitutes. They are substitutes in their ability to provide a given amount of kWh at some date but they also are complements because they cannot be dispatched with the same delays and their operating and installation costs are quite different. For example, for long-duration consumption (the base load), electricity from nuclear plants is cheaper than electricity from gas turbines, but for short-duration consumption (the peak load) the converse is true. Hence, no technology is better than the others in absolute terms. This is because electric energy cannot be stored as such and must be produced at the very moment it is called for. Fortunately, the fuels used to generate electricity are storable, except for wind, although at different costs. One interesting fuel is water. In this paper, we analyze the interaction between an hydroelectric generator and a thermal generator when there exists a technical possibility to pump water up into the reserves of the hydroplant using power produced by the thermal generator.

The observation of real hydrosystems shows that hydrogenerators operate complex systems of dams where the water released and turbinated by an upstream dam can be stored in a downstream dam and either turbinated a second time or pumped to come back into the upstream dam. The economic analysis of such a complex system necessitates to construct a huge model to take into account all the technical restrictions imposed to the quantity of water that can be stored (at maximum and at minimum), released, turbinated, pumped etc. in each dam. The objective of this paper is to qualify how water pumping changes the overall conditions of competition between heterogeneous technologies of generation\(^1\). Consequently, in the following pages, we adopt strongly simplifying hypotheses as compared with the real world. In particular, we omit stock constraints and we focus on flow restraints.

While the electricity industry is commonly viewed as an industry where storage is totally impossible, engineers (and some economists) know that the electricity produced during off-peak periods by fuel generators can be used to store water and this provides extra energy for peak periods\(^2\). According

\(^1\)The basic model of competition between a thermal plant and a hydroplant (without pumping) is given in Crampes and Moreaux (2001).

\(^2\)This pumped storage technique is particularly well suited to nuclear plants that cannot be “turned off” during low demand intervals. By using electricity for pumped storage, the nuclear generators can operate continuously at their most efficient output.
to Eurostat data, pumping represents 5.5% of the generation capacity in the European Union. As shown in Table 1, the energy produced by means of this stored water is only 0.96% of the total production but it is an essential supply because it is called at peak periods, when thermal electricity becomes scarce and expensive. Within an integrated industry, the dispatching of the two types of plants at peak and off-peak periods is just a matter of efficiency. This part of the economic analysis has been clearly documented by Jackson (1973). But with the liberalization of the electricity industry, a new problem arises: how can competition in electricity markets work when hydroplants and thermal plants belong to separate owners? Indeed, the two types of generators are competitors at peak periods and potential partners at off-peak periods. This complex situation has not been addressed yet in the economic literature.

**Table 1**

<table>
<thead>
<tr>
<th>Country</th>
<th>Total production</th>
<th>Hydraulic production</th>
<th>Pump storage share</th>
</tr>
</thead>
<tbody>
<tr>
<td>Austria</td>
<td>64.70</td>
<td>39.90</td>
<td>2.50</td>
</tr>
<tr>
<td>Belgium</td>
<td>86.30</td>
<td>1.60</td>
<td>1.30</td>
</tr>
<tr>
<td>France</td>
<td>571.80</td>
<td>64.90</td>
<td>5.19</td>
</tr>
<tr>
<td>Germany</td>
<td>600.00</td>
<td>27.00</td>
<td>6.40</td>
</tr>
<tr>
<td>Italy</td>
<td>298.90</td>
<td>48.70</td>
<td>7.50</td>
</tr>
<tr>
<td>Norway</td>
<td>110.40</td>
<td>109.30</td>
<td>0.52</td>
</tr>
<tr>
<td>Poland</td>
<td>154.40</td>
<td>3.69</td>
<td>1.61</td>
</tr>
<tr>
<td>Spain</td>
<td>278.10</td>
<td>34.40</td>
<td>2.90</td>
</tr>
<tr>
<td>United Kingdom</td>
<td>400.30</td>
<td>7.60</td>
<td>2.60</td>
</tr>
<tr>
<td>European Union</td>
<td>3170.30</td>
<td>334.80</td>
<td>33.30</td>
</tr>
<tr>
<td>Japan</td>
<td>1111.90</td>
<td>103.60</td>
<td>9.10</td>
</tr>
<tr>
<td>USA</td>
<td>4121.50</td>
<td>299.10</td>
<td>26.60</td>
</tr>
<tr>
<td>Russia</td>
<td>931.00</td>
<td>167.80</td>
<td>1.90</td>
</tr>
</tbody>
</table>


Section 2 presents the features of the model where electricity can be produced either by a thermal plant or by a hydroplant supplied with water by electricity from the thermal plant. In section 3 we expose the characteristics levels”. Jackson (1973) p. 556.

3Production in 1999 was 24.3 TWh out of a total of 2533 TWh for the fifteen countries of the EU.
of the efficient use of the two plants and we show how pump storage creates a strong complementarity between the two types of technologies. Section 4 determines and compares the decisions made by a benevolent planner and by a private integrated monopoly. In section 5, we switch to competition between two private operators, each controlling one technology. Actually, they mainly compete during the periods of peak demand. When the final demand for electricity is low, the hydroplant is the client of the thermal generator. Section 6 concludes.

2 Model setting

We consider a stationary cyclical electricity market which can be supplied by two types of perfectly substitutable generators, a thermal unit and a hydroplant.

Each 24-hour time interval, denoted by $\tau = 1, 2, \ldots$, is made of two periods, the night period labeled $t = 1$ and the day period labeled $t = 2$, of equal duration. Hereafter, we will refer to “off-peak period” for $t = 1$, to “peak-period” for $t = 2$ and to “day” for the 24-hour time interval. Let $q_{\tau t}$ be the quantity consumed in period $t$ of day $\tau$. By a stationary market we first mean that the gross surplus that final users derive from consuming $q_{\tau t}$ depends on period $t$ but not on day $\tau$. Hence, deleting index $\tau$, let $u_t(q_t)$ be the period $t$ gross surplus generated by the consumption of $q_t$.

We assume that for any $t = 1, 2$, the gross surplus function $u_t : R_{++} \to R_+$ is first strictly concave and increasing over some internal $(0, \hat{q}_{tu})$, $0 < \hat{q}_{tu} < +\infty$, next constant over $[\hat{q}_{tu}, +\infty]$, and satisfying the Inada condition, that is:

- $u_t'(q_t) > 0$ and $u_t''(q_t) < 0$, $q_t \in (0, \hat{q}_{tu})$, $t = 1, 2$
- $\lim_{q_t \to 0^+} u_t'(q_t) = +\infty$ and $\lim_{q_t \to \hat{q}_{tu}} u_t'(q_t) = 0$, $t = 1, 2$
- $u_t(q_t) = \bar{u} > 0$, $q_t \in [\hat{q}_{tu}, +\infty]$, $t = 1, 2$

Period 2 is the peak-period, meaning that:

- $\hat{q}_{1u} < \hat{q}_{2u}$
- $u_1(q) < u_2(q)$, $q \in (0, +\infty)$
- $u_1'(q) < u_2'(q)$, $q \in (0, \hat{q}_{2u})$

Let $R_t(q_t) = u'(q_t)q_t$, $t = 1, 2$, denote the revenue function in period $t$ and let $MR_t(q_t) \equiv R_t'(q_t) = u'_t(q_t) + u''_t(q_t)q_t$ be the corresponding marginal revenue function.
revenue function. We first assume that $MR_t$ is strictly decreasing when positive and cannot be positive over disconnected intervals. Thus there must exist $\hat{q}_{tR}, 0 < \hat{q}_{tR} < +\infty, t = 1, 2$, such that

$$MR_t(q_t) \begin{cases} > 0 & , \ q_t \in (0, \hat{q}_{tR}) \\ \leq 0 & , \ q_t \in [\hat{q}_{tR}, +\infty] \end{cases}$$

and:

$$MR_t'(q_t) = 2u_t''(q_t) + u_t'''(q_t)q_t < 0 \ , \ q_t \in (0, \hat{q}_{tR}).$$

Also we assume that, when positive, the marginal revenue in the peak period is higher than the marginal revenue in the off-peak period, for a same consumption level in each period. Thus:

$$\hat{q}_{1R} < \hat{q}_{2R} \quad MR_1(q) < MR_2(q) \quad , \quad q \in (0, \hat{q}_{2R}).$$

We assume that the operating cost function of the thermal plant is the same whatever the period. Let $c(q)$ be the cost function of the thermal plant which is assumed to be of class $C^2$ and strictly increasing and convex over the range of feasible production levels. We assume that there exists an interval $(0, \bar{q})$ such that $u_t(q) - c(q) > 0, \ t = 1, 2$ so that dispatching will always command to have positive generation for final consumers.

Concerning now the hydro-plant system, let us measure water in the dam in terms of energy units. The type of hydro-plant we have in mind is an Alpine mountain system in which the dam is located at some high altitude site whereas the turbine generators are located at a lower altitude site, so that the height of water stock in the dam itself can be neglected. Let $S_{\tau t}$ be the quantity of water available in the dam at the end of period $t$ of day $\tau$.

We assume that there exists no natural water inflow entering the system. All the water available in the dam must have been pumped from some source, using outside electric energy, that is energy generated by the thermal plant. Let $f_{\tau t}$ be the quantity of water pumped by the hydro-producer during period $t$ of day $\tau$. Assuming that the storage capacity of the dam is sufficiently large so that no storage constraint is ever active, assuming there is no loss of water within the system, and denoting by $q_{tH}$ the output of the hydro-plant in period $t$ of day $\tau$, absent any random event, we must have

$$S_{\tau t} \leq \begin{cases} S_{\tau -1, t+1} + f_{\tau t} - q_{tH} & \text{if } \ t = 1 \\ S_{\tau, t-1} + f_{\tau t} - q_{tH} & \text{if } \ t = 2 \end{cases}$$
Let us denote by \( q^T_{t\tau} \) the thermal production of period \( t \) in day \( \tau \) for use within the same period. We limit our attention to cyclical stationary states in which for any \( \tau \) and \( \tau' \) and any \( t \), we have:

\[
\begin{align*}
    f_{\tau t} &= f_{\tau' t}, \\
    q^H_{\tau t} &= q^H_{\tau' t}, \\
    q^T_{\tau t} &= q^T_{\tau' t}, \quad \text{and} \quad S_{\tau t} = S_{\tau' t},
\end{align*}
\]
so that we must have \( q^H_{\tau 1} + q^H_{\tau 2} \leq f_{\tau 1} + f_{\tau -1, 2} = f_{\tau 1} + f_{\tau 2} \). Hence in what follows we drop the index \( \tau \).

Pumping water necessitates more energy than the pumped water can generate. Let \( \alpha f_t, \alpha > 1 \), be the quantity of electricity required to add \( f_t \) to the stock available in the dam at the beginning of the next period. The total production level of the thermal unit is \( q^T_t + \alpha f_t \).

3 Efficient production schemes

When the decision whether to dispatch the thermal and the hydroplants and for how much is taken by one single entity, the mix of hydro and thermal electricity minimizes the total cost of electricity generation. We analyze the efficient production schemes in this section. In the next section, we will consider the final decision on the absolute values of dispatch depending on the objective of the decision-taker, either a public firm that maximizes social welfare or a private monopolist that maximizes profit.

How to produce efficiently a given pair \( (q_1, q_2) \) of electric energy? The efficient dispatch \( \{(q^H_t, q^T_t, f_t), t = 1, 2\} \) is the solution to problem P.1:

\[
\begin{align*}
\text{P.1} \quad \max_{\{q^H_t, q^T_t, f_t, t = 1, 2\}} & - c(q^T_1 + \alpha f_1) - c(q^T_2 + \alpha f_2) \\
\text{s.t.} & \\
q^T_t + q^H_t - q_t & \geq 0, \quad t = 1, 2 \quad \gamma_t \quad (3.1) \\
(f_1 + f_2) - (q^H_1 + q^H_2) & \geq 0 \quad \mu \quad (3.2) \\
q^H_t & \geq 0, \quad q^T_t \geq 0, \quad f_t \geq 0, \quad t = 1, 2 \quad \nu \quad (3.3)
\end{align*}
\]

Constraint (3.1) means that electricity cannot be stored and (3.2) recalls that the whole hydroresource comes from pumping. The first order conditions that characterize the solution to problem P.1 are detailed in the Appendix. From these conditions, we deduce that cost minimization commands constraints (3.1) and (3.2) to be binding. Actually, it would be wasteful to generate more energy than what is needed (constraint (3.1)) and to pump
more water than what will be turbinated (constraint (3.2)). Hence, in what
follows, we refer to (3.1) and (3.2) as equalities. Also, an immediate implica-
tion of the basic laws of thermodynamics, is that transforming first electricity
into water in the dam and next water in the dam into electricity, both within
the same period, is pure energy waste. Thus having both $q^H_t$ and $f_t$ strictly
positive cannot be a component of a cost minimizing policy. Summing up:

**Lemma 1** Assume that $q_t > 0$, then cost minimization implies that:

- $a$ - both constraints (3.1) and (3.2) must be satisfied as equalities;
- $b$ - $q^H_t$ and $f_t$ cannot be both strictly positive.

**Proof** see Appendix

An immediate implication of Lemma 1-b is that the electricity coming
from turbinated water within any period $t$ is actually coming from pumped
water within period $t - 1$. But, because the marginal cost of the thermal
plant is increasing, then the period within which water has to be pumped
is that period during which the consumption of the final users is the lowest
one. Hence:

**Lemma 2** Whatever $t$ and $t'$, $(t, t' = 1, 2$ and $t \neq t')$ then $q_t \geq q'_t > 0$
implies that $q^H_{t'} = 0$.

**Proof** See Appendix.

A straightforward implication of Lemma 2 is that if $q_t = q_{t'}$, then $q^H_{t'} =
q^H_t = 0$ and consequently $f_{t'} = f_t = 0$. Actually, this specific result can be
generalized in the following way. In the present setting, the hydro-system is
a mere storage device or transfer device of the energy produced at one period
to the next at a cost represented by the fraction of the energy lost during the
transfer. Hence, this apparatus is to be used if and only if, without transfer,
the marginal cost differential resulting from the production of quantity $q_t$
exclusively during period $t$ is higher than the marginal loss implied by the
transfer. This is formally established in the following Lemma.

**Lemma 3** Suppose $q_t \geq q_{t'} > 0$, $t, t' = 1, 2$, and $t \neq t'$. Then $c'(q_t) \leq$
$c'(q_{t'})$ is a necessary and sufficient condition for $f_t = f_{t'} = 0$.

**Proof of Lemma 3** see the Appendix.
In order to illustrate how to determine the types of production profiles $(q_t, q_{t'})$ having to be supplied with hydrogenation and thermal generation and those having to be supplied only by thermal generation, let us consider Figure 1. Let us assume that the marginal cost function is a linear function with $c'(q) \equiv \lim_{q \to 0} c'(q) > 0$. In the left hand side quadrant, production levels $q$ are measured along the vertical axis and the marginal costs $c'$ and $\alpha c'$ are measured along the horizontal axis. In the right hand side quadrant, $q_t$ is measured along the vertical axis and $q_{t'}$ along the horizontal axis. Let $q$ be that value of $q$ such that $c'(q) = \alpha c(0)$. For any $q > q_t$ let us define $q_{tm}(q_{t'})$ as this value of $q_t$ solving $c'(q_t) = c'(q_{t'})/\alpha$, and symmetrically for any $q_t > q$ let us define $q_{tm}(q_{t'})$ as this value of $q_{t'}$ solving $c'(q_{t'}) = c'(q_t)/\alpha$. For example in Figure 1 let us start from the value $\hat{q}_{t'}$ of $q_{t'}$, $(\hat{q}_{t'} > q)$, then

$D'$ in the left hand side quadrant is the point $(\alpha c'(\hat{q}_{t'}), \hat{q}_{t'})$,

$D''$ within the same quadrant is the point $(c'(q_{tm}(\hat{q}_{t'})), q_{tm}(\hat{q}_{t'}))$,

$D'''$ in the right hand side quadrant is the point $(q_{tm}(\hat{q}_{t'}), \hat{q}_{t'})$.

Now let us start from the value $\hat{q}_t$ of $q_t$, $(\hat{q}_t > q)$. Then

$C'$ in the left hand side quadrant is the point $(c'(\hat{q}_t), \hat{q}_t)$,

$C''$ within the same quadrant is the point $(\alpha c'(q_{tm}(\hat{q}_t)), q_{tm}(\hat{q}_t))$,

$C'''$ in the right hand side quadrant is the point $(q_{tm}(\hat{q}_t), \hat{q}_t)$.

Let $q_t$ be any quantity of energy to be delivered in period $t$ and $q_{t'} > q_{tm}(q_t)$ be some quantity to be delivered in period $t'$. Assume first that both $q_t$ and $q_{t'}$ are produced by the thermal plant within the delivery period. Then the thermal marginal cost of $q_{t'}$, that is $c'(q_{t'})$, is higher than the marginal cost of $\alpha$ additional units of energy produced in period $t$ in the thermal plant, pumped into the dam and turbinated in period $t'$ to deliver one unit of energy, which amounts to $\alpha c'(q_t)$. Thus for profiles $(q_t, q_{t'})$ located above the line $q_{tm}(q_{t'})$, hydrogenation has to be used. The same argument clearly holds for profiles $(q_t, q_{t''})$ located to the right of the $q_{tm}(q_t)$ line. For profiles located below $q_{tm}(q_{t'})$ and above $q_{tm}(q_t)$, the best solution is to use the thermal unit exclusively.

Clearly for $\alpha = +\infty$, the pumping cost in terms of energy lost is infinite so that no intertemporal smoothing device is available: $q_{tm}$ is the vertical axis and $q_{tm}$ the horizontal axis. At the other end of the spectrum, $\alpha = 1$, transferring energy from some period to the next is costless, and the only profiles for which hydrogenation is not used are those profiles $(q_t, q_{t'})$ such that $q_t = q_{t'}$. Then $q_{tm}(q_{t'})$ and $q_{tm}(q_t)$ are both the $45^\circ$ line. In such a case, whatever $(q_t, q_{t'})$, we must have $q_t^T = q_{t'}^T = (q_t + q_{t'})/2$, $q_{t'}^H = 0$ if $q_t \leq (q_t + q_{t'})/2$, and $q_t^H = q_t - (q_t + q_{t'})/2$, if $q_t > (q_t + q_{t'})/2$, $t, t' = 1, 2$ and
The exact shape of the \(NH\) region depends on the properties of the marginal cost function. In Figure 1, we have assumed that marginal cost is linear, \(c'(q) = c_0 + cq\). Under this specification, it is easy to derive \(q_{tm}(q') = (\alpha - 1)\frac{c_0}{c} + \alpha q'\). Therefore, the \(NH\) region expands with \(\alpha\) and \(c_0\) and shrinks with \(c\).

\[ t \neq t'. \]

We see that if \(\alpha = 1\) the \(NH\) region vanishes since electricity can be transferred from one period to the other costless thanks to the hydroplant.

**Figure 1:** Determination of the production profiles \((q_t, q_v)\) to be produced with and without hydrogeneration

\[ NB: A = c'(0), B = \alpha c'(0), C = c'(\dot{q}_t) = \alpha c'(q_{tm}(\dot{q}_t)), D = \alpha c'(\dot{q}_v) = c'(q_{tm}(\dot{q}_v)), \dot{q} = q_{tm}(0) = q_{vm}(0). \]
When $\alpha = +\infty$ if the thermal cost is linear ($c_0 > 0$, $c = 0$), it is always more efficient to rely only on instantaneous thermal generation rather than to use the storage possibility: the whole right quadrant is a $NH$ zone. When $1 < \alpha < +\infty$ and the thermal cost is purely quadratic ($c_0 = 0$, $c > 0$) the $NH$ zone is a cone, the frontier of which only depends on the efficiency parameter $\alpha$.

4 Integrated management

In this section, we successively consider the first best allocation that is the dispatch that maximizes welfare and the dispatch chosen by a private monopolist who controls the two plants. They only differ in the benefit function to maximize. They face the same technical constraints.

4.1 First best dispatch

The social planner solves the following problem$^5$:

$$\begin{align*}
P.2 \max_{\{q^H_t, q^T_t, f_t\}, t=1,2} & \sum_{t=1}^{2} \{u_t(q^T_t + q^H_t) - c(q^T_t + \alpha f_t)\} \\
\text{s.t.} \ (3.2) \ \text{and} \ (3.3).
\end{align*}$$

The first order conditions of $P.2$ are

$$
\begin{align*}
q^T_t &: u'_t(q^T_t + q^H_t) = c'(q^T_t + \alpha f_t) - \nu^T_t \quad t = 1, 2 \\
q^H_t &: u'_t(q^T_t + q^H_t) = \mu - \nu^H_t \quad t = 1, 2 \\
f_t &: \alpha c'(q^T_t + \alpha f_t) = \mu + \nu^T_t \quad t = 1, 2
\end{align*}
$$

and the complementary slackness conditions (A5)-(A8) in the Appendix.

In section 2, we have assumed that $t = 2$ is the peak period. Intuition suggests that $q_2 > q_1$. This is proved in the Appendix. Since optimality implies efficiency, then by Lemma 2, we know that $q^H_1 = 0$. Consequently, it is straightforward to obtain $f_2 = 0^6$. Indeed, it would be inefficient to pump

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$^5$To simplify the problem, we already assume that (3.1) is an equality and we write the sum of the quantities generated by the two plants as the argument of the gross surplus functions.

$^6$More formally, by (4.2) we have $\mu > 0$ so that $f_1 + f_2 = q^H_2$ by (A5) in the Appendix. Now, assume that $q^H_2 > 0$. By Lemma 1.b we would have $q^H_2 = 0$, hence a contradiction.
water at the peak period with the objective to keep it unused at the next period. The only question that is not yet solved is to know whether $f_1 = q_2^H$ is strictly positive or is nil.

Let $q_{ti}, t = 1, 2$ be the quantities produced and consumed at period $t$ in a purely thermal system, that is in a system where $\alpha = +\infty$. In such a system, the optimal quantities, denoted by $q_{ui}, t = 1, 2$, are those values of $q_i$ solving:

$$u'_t(q_{ti}) = c'(q_{ti}) \quad t = 1, 2 \quad (4.4)$$

We can now characterize the optimal policy in the complete system by comparison with what would prevail in a pure thermal system.

**Proposition 1** The first best dispatch must satisfy:

- if $u'_2(q_{2i}^u) > \alpha c'(q_{1i}^u)$, then:
  $$f_1 = q_2^H > 0 \quad \text{and} \quad f_2 = q_1^H = 0$$
  $$u'_1(q_1^T + \alpha q_2^H) = c'(q_1^T + \alpha q_2^H)$$
  $$u'_2(q_2^T + q_2^H) = c'(q_2^T) = \alpha c'(q_1^T + \alpha q_2^H)$$

- if $u'_2(q_{2i}^u) \leq \alpha c'(q_{1i}^u)$, then:
  $$f_1 = f_2 = q_1^H = q_2^H = 0 \quad \text{and} \quad q_t^T = q_t^u, \ t = 1, 2$$

**Proof of Proposition 1:**

As we saw in Lemma 3 and in Figure 1, for a given pair of final consumptions, depending on the cost function and the loss coefficient $\alpha$, the optimal solution will be either without hydrogeneration or a mix of hydro and thermal generation. The higher $\alpha$, the more likely the no-hydro solution.

Starting from the only-thermal solution $(q_{1i}^u, q_{2i}^u)$, we can define the threshold value

$$\alpha_u = \frac{c'(q_{2i}^u)}{c'(q_{1i}^u)} \quad (4.5)$$

such that if $\alpha \geq \alpha_u$ the no-hydro dispatch is optimal and if $\alpha < \alpha_u$, the best dispatch commands to mix the two technologies. Clearly if $\alpha c'(q_{1i}^u) < c'(q_{2i}^u)$, consuming the same quantity at each period but producing some part $q_2^H$ of $q_{2i}^u$ in period 1, that is producing thermally $q_{1i}^u + \alpha q_2^H$ in period 1 and $q_{2i}^u - q_2^H$ in period 2 would reduce the total cost provided that $q_2^H$ is not too large. This is the case illustrated in Figure 2 for $\alpha = \alpha_1$ and it corresponds to the first part of Proposition 1. Symmetrically, $\alpha_2 c'(q_{1i}^u) > c'(q_{2i}^u)$ so that, it is better not to use the hydro system. Equivalently for a given loss index $\alpha$
and a given cost function \( c \), if the difference between \( u_1 \) and \( u_2 \) and \( u'_1 \) and \( u'_2 \) is not too large so that \( q_{2i}^u - q_{1i}^u \) is small, then the hydro transfer system is useless; and for a given loss index \( \alpha \) and a given utility function, if the marginal cost is slowly increasing, again the hydro system is not used. This case corresponds to the second part of Proposition 1.

![Figure 2: Deviations out of the all-thermal solution](image-url)
The first best allocation is illustrated in Figure 3. The solution in a temporally isolated pure thermal system is illustrated by $A_t$, $t = 1, 2$, for period $t$. Note that we have $q_1^T < q_1^u$, that is the first best consumption in period 1 is lower than the consumption in period 1 in the temporally isolated pure thermal system, whereas the consumption in period 2, the peak period, is higher $q_2^T + q_2^H > q_2^u$.

But the thermal production in period 1 is higher, and in period 2 is lower than in the pure thermal system. Compared to the pure thermal system there is a welfare loss in period 1, measured by the shaded area 1, more than compensated by a welfare gain in period 2, measured by the shaded area 2. If marginal utility is very inelastic, welfare gains from pumping are essentially technical. Otherwise, consumers incur a loss at night but their utility is increased at day.
4.2 The private monopoly

When the two plants are operated by a unique private firm, they are operated efficiently because the firm is minimizing its cost. Thus the difference between the optimally managed system and the unregulated private monopoly lies in the different objective functions each one wants to maximize. The program solved by the monopoly is like program P.2, substituting the revenue functions \( R_t \) for the gross surplus functions \( u_t \) at each period \( t = 1, 2 \). Let us call P.3 this program. The first order conditions are:

\[
\begin{align*}
    u_t'(q_t^T + q_t^H) + u_t''(q_t^T + q_t^H)[q_t^T + q_t^H] &= \eta(q_t^T + \alpha f_t) - \nu_t^H, \quad t = 1, 2 \\
    u_t'(q_t^T + q_t^H) + u_t''(q_t^T + q_t^H)[q_t^T + q_t^H] &= \mu - \nu_t^H, \quad t = 1, 2
\end{align*}
\]

(4.6) (4.7)

together with (4.3) and the complementary slackness conditions (A.5)-(A.8) in the Appendix.

By comparing the two dispatches solving respectively P.2 and P.3, we mean:

- First compare the consumption levels, period by period, in each setting
- Second compare the types of production mix, thermal and hydro energies, used in each case.

Concerning the first point, the monopoly effect is the standard effect, that is a consumption level in each period lower than the optimal consumption level.

As regards the second it must be clear that anything may happen, that is:

- either the same type of mix in the two settings, both thermal and hydro energies in each one, or thermal energy only in each one;

- or different types of mix, either both energies at the optimum and only thermal under monopoly, or only thermal at the optimum but both energies under monopoly.

The reasons why we may have all these cases is illustrated in Figure 4 and with the following example.
Figure 4: First best vs. private monopoly

Consider first Figure 4 which is nothing but the right hand side quadrant of Figure 1 with $q_2$, the peak-period production level, measured along the vertical axis, and $q_1$, the off-peak production level, measured along the horizontal axis. Since both optimal and monopolistic peak period production levels are higher than the off-peak period ones, both optimal and monopolistic production profiles are located above the 45° line.

In Figure 4 each arrow illustrates a possible move from the optimal production profile toward the production profile chosen by the monopoly. With arrows 1 and 2 the types of production mix are the same in the two settings, using the two kinds of energy when moving along Arrow 1, using thermal energy only when moving along Arrow 2. With Arrows 3 and 4, the production mix changes when passing from the optimal profile to the monopoly profile. When moving along Arrow 3, thermal energy only is used at the optimum while both energies are used by the monopoly. On the contrary, when moving along Arrow 4, both energies are used at the optimum while thermal energy only is used by the monopoly.

The following example shows that any one among the moves just listed is possible.

Assume that $u'_t(q_t) = a_t - b_t q_t$ and $c'(q_t) = c q_t$ , $t = 1, 2$ where $a_2 > a_1 > c$.

The first-best non-hydro solution is given by $u'_t(q_t) = c q_t$ , that is
The non-hydro solution when outputs are chosen by a private monopoly is the solution to \( u_t'(q_t) + q_t u_t''(q_t) = c q_t \), that is

\[
q_t^R = \frac{a_t}{2b_t + c} \quad t = 1, 2
\]  

(4.9)

With these values, it is easy to compute

\[
\alpha_u = \frac{a_2}{b_2 + c} \frac{b_1 + c}{a_1} \quad \text{and} \quad \alpha_R = \frac{a_2}{2b_2 + c} \frac{2b_1 + c}{a_1}
\]

which represent the threshold value of the loss due to pumping in the first best dispatch and in the monopoly dispatch respectively\(^7\). We see that \( \alpha_u \not\leq \alpha_R \) according to \( b_2 \not\leq b_1 \).

Since period 2 is the peak-period, we can assume that \( b_2 > b_1 \), denoting a demand less reactive to price variation. It results that

- if \( \alpha > \alpha_u \), the hydroplant is used neither by the social planner, nor by the monopolist,
- if \( \alpha_u > \alpha > \alpha_R \), the monopolist does not use the hydroplant while it would be optimal to use it,
- if \( \alpha_R > \alpha \), the hydroplant is used both by the social planner, and by the monopoly.

Hereafter, we compare the optimal despatch and the monopoly’s despatch when \( \alpha \) is small enough for pumping to be used in both cases.

For \( \alpha < \alpha_u \), from Proposition 1 we can write the first best dispatch as

\[\]  

\(7\)The critical value of \( \alpha \) corresponding to first best is defined in (4.5). The critical value that corresponds to the monopoly case is defined by \( \alpha_R = \frac{c'(q_t^R)}{c'(q_t^R)} \), where \( q_t^R \) is the value of \( q_t \) solving \( u_t'(q) + q u_t''(q) = c'(q) \), \( t = 1, 2 \).
\[ q_2^{Hu} = \frac{(\alpha_u - \alpha)a_1a_2}{\alpha^2a_2b_1 + \alpha_u b_2a_1} \]

\[ q_1^{Tu} = \frac{\alpha_u a_1b_2 + \alpha^2a_2b_1 - \alpha \alpha a_2(\alpha_u - \alpha)}{(b_1 + c)(\alpha_u a_1b_2 + \alpha^2a_2b_1)} = q_1^u - \frac{\alpha c}{b_1 + c} q_2^{Hu} \]

\[ q_2^{Tu} = \frac{\alpha a_2[a_1b_2 + \alpha a_2b_1]}{(b_2 + c)(\alpha_u a_1b_2 + \alpha^2a_2b_1)} = q_2^u - \frac{b_2}{b_2 + c} q_2^{Hu} \]

As shown in figure 3, we observe that

\[ q_1^{Tu} + \alpha q_2^{Hu} = q_1^u + \alpha \frac{b_1}{b_1 + c} q_2^{Hu} > q_1^u > q_1^{Tu} \]

and

\[ q_2^{Tu} + q_2^{Hu} = q_2^u + \frac{c}{b_2 + c} + q_2^{Hu} > q_2^u > q_2^{Tu} \]

which means that at each period, the total production of electricity is larger than if the pumping plant were not available. At the off-peak period (\( t = 1 \)) final consumption \( q_1^{Tu} \) is lowered by pumping but at the peak period (\( t = 2 \)) final consumption \( q_2^{Tu} + q_2^{Hu} \) is higher thanks to the extra energy provided by the hydroplant.

The same comments apply to the monopoly dispatch when \( \alpha_R > \alpha \):

\[ q_2^{HR} = \frac{(\alpha_R - \alpha)a_1a_2}{2[\alpha^2a_2b_1 + \alpha_R a_1b_2]} \]

\[ q_1^{TR} = q_1^R - \frac{\alpha c}{2b_1 + c} q_2^{HR} \]

\[ q_2^{TR} = q_2^R - \frac{2b_2}{2b_2 + c} q_2^{HR} \]

If we compare the monopoly dispatch with the first best when \( \alpha_u > \alpha_R > \alpha \), it is straightforward to check that the monopoly does not use the hydroplant sufficiently as compared with first best: \( q_2^{HR} < q_2^{Hu} \). This is trivially true when \( \alpha_u > \alpha_R = \alpha \) since \( q_2^{Hu} > 0 \) while \( q_2^{HR} = 0 \).

It remains to compare the intensity of use of the thermal plant. We can compute that

\[ q_1^{Tu} + \alpha q_2^{Hu} > q_1^{TR} + \alpha q_2^{HR} \quad (4.10) \]
and

\[ q_T^{nw} > q_R^{TR} \]  

(4.11)

In other words, the possibility to combine the two technologies does not modify the behavior of the private monopoly: its thermal plant is used below the optimal level.

5 Cournot Competition

We assume in this section that the thermal plant and the hydro plant are under the control of different enterprises and we consider a “day-ahead” wholesale market for electricity in which:

i) the two agents announce to an independent Market Operator (MO thereafter) the quantities they intend to supply and/or demand in each period of the following day; in other words, they play an open loop game.

ii) taking into account the demand functions of the final users the IMO computes the price that clears the market in each period.

We neglect the markets where the price at which the electricity sold to the hydro producer or sold by the hydro producer is fixed by administrative rules as it is the case for generation from renewable resources in some countries. Therefore, the energy price is the same for final use and for intermediary use.

As regards quantity, we assume that \( H \) and \( T \) play an open loop Cournot game to supply final consumers. By contrast, alternative hypothesis are possible concerning the volume of pumped water: i) it is fixed by the social planner; ii) it is fixed by the thermal plant operator (via the quantity of electricity sold to \( H \)); iii) it is fixed by the hydroplant operator who is eligible to demand electricity on the wholesale market. In order to simplify the proofs of the different possibilities, we will assume that \( f_2 = 0 \) in all cases.

5.1 The Cournot game

Final consumers buy \( q_t \) at period \( t \). Since they are price takers, their demand is the solution to

\[ \max_{q_t} u_t(q_t) - p_t q_t \text{ where } q_t = q_t^H + q_t^T, \quad t = 1, 2. \]
From the first order condition, we can write the inverse demand function as
\[ \tilde{p}_t(q_t) = u'_t(q^H_t + q^T_t) \quad t = 1, 2 \] (5.1)

Given this price relation and the quantities chosen by the competitor, each duopolist maximizes its profits. The thermal generator solves

\[
\max_{q^T_1, q^T_2} \sum_{t=1}^{2} \left[ \tilde{p}_t(q^H_t + q^T_t)(q^T_t + \alpha f_t) - c(q^T_t + \alpha f_t) \right]
\] (5.2)

In fact, to pump \( f_t \) of water into the hydroproducer’s reservoir, a quantity of electricity \( \alpha f_t \) must be added to the demand from final consumers \( q^T_t \). Therefore, the total output of the thermal unit is \( q^T_t + \alpha f_t \).

Assuming that \( q^T_t > 0 \) at both periods, the first order conditions to solve (5.2) are

\[ \tilde{p}_t + (q^T_t + \alpha f_t)\tilde{p}'_t - c'(q^T_t + \alpha f_t) = 0 \quad t = 1, 2 \] (5.3)

from which we can write the best response function of the thermal plant at period \( t \) as

\[ q^T_t = Q^T_t(q^H_t, f_t) \quad t = 1, 2 \] (5.4)

This function is decreasing both with \( q^H_t \) and \( f_t \).

As regards the final market, the problem of the hydroproducer is

\[
\max_{q^H_1, q^H_2} \sum_{t=1}^{2} \tilde{p}_t(q^H_t + q^T_t)(q^H_t - \alpha f_t)
\]

s.t. \((f_1 + f_2) - (q^H_1 + q^H_2) \geq 0 \quad \mu \)

\[ q^H_t \geq 0 \quad t = 1, 2 \quad \nu \]

The first order conditions are

\[ \tilde{p}_t + (q^H_t - \alpha f_t)\tilde{p}'_t - \mu + \nu H^H_t = 0 \quad t = 1, 2 \] (5.5)

The solution to the hydroplant problem is

\[ q^H_t = Q^H_t(q^T_t, q^T_t, f_1) \quad t = 1, 2 \]

Depending on the value of the thermal production for final consumers at both dates and the value of \( f_1 \), we can have one of the following possibilities:

i) \( q^H_1 = q^H_2 = 0 \),
ii) \( q^H_2 > q^H_1 = 0 \),
iii) \( q^H_2 > q^H_1 > 0 \)
5.2 The inflow is exogenous

Assume first that $f_1$ is fixed neither by $H$ nor by $T$. For instance, it is imposed by the central planner in order to maximise welfare knowing that firms will ultimately play a Cournot game in the consumers’ market at each period.

This corresponds to the case analyzed in Crampes and Moreaux (2001) with the additional feature that the thermal plant incurs a cost and receives the off-peak-period market price to provide water to its competitor.

To explore this case, one must first solve the market equilibrium at periods 1 and 2, obtaining the Cournot outputs parametrized by $f_1$:

$$q^{Tc}_1(f_1), q^{Tc}_2(f_1), q^{Hc}_1(f_1), q^{Hc}_2(f_1).$$

Then, the social planner solves

$$\max_{f_1} \max_{t=1,2} \{u_t(q^{Tc}_t + q^{Hc}_t) - c(q^{Tc}_t + \alpha f_t)\}$$

s.t. $q^{Tc}_t \geq 0, \quad q^{Hc}_t \geq 0, \quad t = 1, 2$

$$\pi^H_t(q^{Hc}_1, q^{Hc}_2) = 0, \quad \pi^T_t(q^{Tc}_1, q^{Tc}_2) \geq 0, \quad t = 1, 2$$

where $f_2 = 0$.

TBC

5.3 The hydroproducer is an eligible consumer

In this legal configuration, $H$ chooses $f_1 \geq 0$ and we must add condition

$$-\alpha \tilde{p}_1 + \mu + \nu^f_t = 0$$

(5.6)

to (5.5).

Note first that $\tilde{p}_1 = \tilde{p}_2$ would be a necessary condition for having both $f_1 > 0$ and $f_2 > 0$. Keeping the hypothesis that period 2 is the peak-period, we can assume that $\tilde{p}_2 > \tilde{p}_1$ at equilibrium, so that $f_2 = 0$ for sure. Indeed, pumping at period 1 and pumping at period 2 are perfect substitutes for the hydrogenerator and the best choice is a corner solution: all the electricity to store water will be bought at the off-peak period. Note also that $f_1 = 0$
would mean that the hydroplant would remain out of the market. From now, we assume that \( f_1 > 0 \), so that \( \nu^H_1 = 0 \).

With a given quantity \( f_1 \) in its reservoir, the hydro generator has to decide the allocation between the two periods. We successively consider the case where \( q^H_1 > 0 \) and the case where \( q^H_1 = 0 \).

### 5.3.1 Selling hydroelectricity at the off-peak period

When \( q^H_1 > 0 \), the arbitrage conditions (5.5) and (5.6) reads

\[
\tilde{p}_1 + (q^H_1 - \alpha f_1)\tilde{p}'_1 = \alpha \tilde{p}_1 = \tilde{p}_2 + q^H_2 \tilde{p}'_2 
\]

(5.7)

where \( f_1 = q^H_1 + q^H_2 \).

The second equality in (5.7) simply states that at period 2 firm \( H \) produces up to the point where the last kWh gives a marginal revenue (from the demand not served by firm \( T \)) equal to the cost of pumping water with that purpose at period 1.

The first equality can be rewritten as

\[
\tilde{p}_1 + q^H_1 \tilde{p}'_1 = \alpha [\tilde{p}_1 + f_1 \tilde{p}'_1].
\]

At period 1, the marginal revenue from sales to final consumers is equal to the marginal cost of buying electricity to pump water, taking into account the decrease in price due to the demand for pumping (that includes the demand with a view to produce electricity at period 2).

Solving (5.7) gives the best response functions

\[
q^H_t = Q^H_t(q^H_1, q^H_2), \quad t = 1, 2
\]

(5.8)

Using \( f_1 = q^H_1 + q^H_2 \) and \( f_2 = 0 \), we see that the best response of the thermal plant (5.4) are

\[
q^T_1 = Q^T_1(q^H_1, q^H_2 + q^H_1) = \tilde{Q}^T_1(q^H_1, q^H_2)
\]

\[
q^T_2 = Q^T_2(q^H_2, 0) = \tilde{Q}^T_2(q^H_2)
\]

Intersecting these best response functions with (5.8) gives the Cournot equilibrium, if it exists.

What is the intuition that explains that firm \( H \) is buying and selling electricity at the same period, namely the off-peak period? Because \( \alpha > 1 \),
this operation obviously creates a loss. But selling electricity at period 1 has
the advantage to decrease the price at which electricity is bought to pump
water that will be turbinated to sell electricity at period 2. From the first
equality in (5.7), we see that the net gain from selling one unit rather than
0 at period 1 (i.e. starting from \( q_1^H = 0, f_1 = q_2^H \)) is

\[
(1 - \alpha)\tilde{p}_1(q_1^T) - \alpha q_2^H \tilde{p}_1'(q_1^T)
\]

If the hydrogenerator expects to sell a large \( q_2^H \) and/or if the price function
is very reactive at period 1 (if \( \tilde{p}_1' < 0 \) is large in absolute value), we see that
the second term can overweight the first one, so that the best choice of firm
\( H \) is to sell electricity at the off-peak period \( (q_1^H > 0) \) and to buy at the same
period \( (f_1 > 0) \).

Consider the following elementary illustration. Let \( p_1 = 100 - q_1 \), \( p_2 = 
200 - q_2 \), \( c(q) = q^2/2 \) and \( \alpha = 1.28 \). Then the Cournot equilibrium gives\(^8\)

\[
q_1^T = 14 \quad \text{,} \quad q_1^H = 4 \quad \text{,} \quad q_2^T = 61 \quad \text{,} \quad q_2^H = 17
\]

The prices are \( p_1 = 82, \ p_2 = 122 \). Then, the hydro firm buys \( 1.28(4 + 17) \simeq 27 \) MWh at period 1. Its total profit is \( 82(4 - 27) + 122 \times 17 = 
-1886 + 2074 = 188 \). All things equal, assume now that the hydroproducer
only buys \( 1.28 \times 17 \simeq 21.8 \) MWh at period 1 and sells \( q_1^H = 0 \). The resulting
price is \( p_1 = 100 - 14 = 86 \) so that the electricity bill of firm \( H \) is \( 86 \times 21.8 = 
1874 < 1886 \). As the revenue at period 2 remains \( p_2 q_2^H = 2074 \), we see that
\( q_1^H = 4 \) is more profitable than \( q_1^H = 0 \).

5.3.2 No hydroelectricity sales at the off-peak period

Let us consider the equilibria where \( q_1^H = 0 \), so that \( f_1 = q_2^H > 0 \).

We have

- for \( T \)

\[
\tilde{p}_1(q_1^T) + (q_1^T + \alpha q_2^H)\tilde{p}_1'(q_1^T) - c'(q_1^T + \alpha q_2^H) = 0 \tag{5.9}
\]

\[
\tilde{p}_2(q_2^T + q_2^H) + q_2^T \tilde{p}_2'(q_2^T + q_2^H) - c'(q_2^T) = 0 \tag{5.10}
\]

- for \( H \)

\[
\tilde{p}_2(q_2^T + q_2^H) + q_2^H \tilde{p}_2'(q_2^T + q_2^H) - \alpha \tilde{p}_1(q_1^T) = 0 \tag{5.11}
\]

\(^8\)The quantities are integer approximations.
Figure 4 depicts the best choice of the thermal producer at periods 1 and 2, given $q_2^H$. At the off-peak period, the thermal producer faces total demand $u_1^{-1}(p_1) + \alpha q_2^H$ which results in a discontinuous marginal revenue function depicted by the two-piece bold curve $MR_1(q_1)$. Actually, the intercept of the final demand curve with the vertical axis $p_1(0)$ can be viewed as a price cap for all buyers, including intermediate buyers. Therefore, when $q_1 \leq \alpha q_2^H, MR_1 = p_1(0)$. At $q_1 = \alpha q_2^H$, marginal revenue jumps down since it is now derived from the demand of both final users and firm $H$. In Figure 4, we assume that the intercept of marginal revenue and marginal cost results in $Q_T^T(q_2^H) > 0$. But, shifting upward the marginal cost function, it is easy to see that the best choice of firm $T$ can be $Q_T^T(q_2^H) = 0$, which means that the final users receive nothing. In this case, the hydroplant can receive exactly what it demands, or it can be rationned, receiving less than $\alpha q_2^H$. At the peak-period (see the right panel in Figure 4) as the hydro firm sells $q_2^H$, the marginal revenue of firm $T$ is derived from the residual demand $u_2^{-1}(p_2) - q_2^H$. The best choice of firm $T$ is $Q_T^T(q_2^H)$ at the intersection of marginal cost and the residual marginal revenue.

![Figure 4: Best responses of the thermal plant](image)

The first order condition of firm $H$ (5.12) recalls that to sell one extra MWh at period 2 the hydroproducer has to buy $\alpha$ MWh at period 1.
Therefore, the marginal cost is $\alpha \tilde{p}_1$. And since $\tilde{p}_1$ depends on $q^T_1$, the best responses of firm $H$ are $q^H_1 \equiv 0$ and $q^H_2 = Q^H_2(q^T_1, q^T_2)$ where $\partial q^H_2 / \partial q^T_1 < 0$ and $\partial q^H_2 / \partial q^T_1 > 0$.

5.4 The thermal producer controls its total output

Suppose now that the thermal firm decides on the total quantity it produces $y^T_t \equiv q^T_t + \alpha f_t$.

Instead of (5.2) the problem it solves is now

$$\max_{y^T_1, y^T_2} \sum_{t=1}^L \{ \tilde{p}_t(q^H_t + y^T_t - \alpha f_t) - c(y^T_t) \}$$

which shows that the hydroproducer receives the residual quantity not bought by final consumers $\alpha f_t = y^T_t - q^T_t$. As compared with the former section, $H$ has lost the control of the quantity; but it still has some control on the price.

The problem of the hydroplant is

$$\max_{1^H, q^H_2} \sum_{t=1}^2 \{ \tilde{p}_t(q^H_t + y^T_t - \alpha f_t)(q^H_t - \alpha f_t) \}$$

s.t. $(f_1 + f_2) - (q^H_1 + q^H_2) \geq 0$ \hspace{1cm} $\mu$ \hspace{1cm} $\nu$

$f_t \geq 0$, $q^H_t \geq 0$

The first order conditions are (5.5) and

$$-\alpha \tilde{p}_t - \alpha \tilde{p}_t(q^H_t - \alpha f_t) + \mu + \nu = 0 \hspace{1cm} (5.12)$$

which shows that, for $f_t > 0$, contrary to what we had in the former case (see (5.6)), the marginal value of the stored water is smaller than its marginal cost: $\frac{\mu}{\alpha} < \tilde{p}_t$.

This has important consequences on feasible outputs. Suppose that $q^H_t$ and $f_t > 0$. Then, from (5.5) and (5.12), we have

$$\tilde{p}_t + \tilde{p}_t(q^H_t - \alpha f_t) = \mu = \alpha [\tilde{p}_t + \tilde{p}_t(q^H_t - \alpha f_t)]$$

This is possible only if

• either $\alpha = 1$
or $\tilde{p}_t + \tilde{p}'_t(q^H_t - \alpha f_t) = 0$, $t = 1, 2$.

In the second case, we have $\mu = 0$. In other words, the stored water has a zero marginal value.

A corollary is that $q^H_t > 0, f_t > 0$ is not possible when $\alpha > 1$. As we have seen, when $\alpha > 1, (q^H_t > 0, f_t > 0)$ entails

$$\tilde{p}_t + \tilde{p}'_t(q^H_t - \alpha f_t) = 0$$

which requires $q^H_t - \alpha f_t > 0$ and, since $\alpha > 1$, $q^H_t > f_t$. This obviously violates the constraint

$$q^H_1 + q^H_2 \leq f_1 + f_2$$

Therefore, in this specification of the model, we cannot have equilibria with $q^H_t$ and $f_t$ both positive.

6 Conclusions

Because hydroplants are more flexible than thermal plants to produce electricity, they play a very important role at peak hours. Indeed, in countries with hydro resources, the marginal technology (that is the last technology in merit order called to match demand) is very often the hydroelectricity technology during peak demand periods. The resource used at peak periods can come from pump storage, a technique that creates complementarity between off-peak thermal electricity and peak hydroelectricity. In this paper, we have analyzed this problem in two different frameworks: one is the joint decision framework where one single decision maker (either public or private) chooses the whole dispatch for the two types of plants for two periods within a steady state regime. In the second one, the two types of technology are under the control of separate private agents. In the first case, efficiency commands to sell no electricity during the (off-peak) period of pumping. In the second case, it appears that at equilibrium, the hydroplant can be better off when selling electricity and pumping simultaneously. This costly decision allows to decrease the marginal cost (that is the off-peak price) of the peak MWh.

The model could be extended in several directions:

- hydro capacity constraints
- restrictions to thermal flexibility (ramping rates, warming-up)
- more competition with several firms, each controlling both technologies
- random natural inflows.
References


Appendix

A.1 Characterization of the solution to problem P.1

Let LP.1 be the Lagrange function of P.1

$$\text{LP.1} = \sum_{t=1}^{2} \left\{ -c(q_t^T + \alpha f_t) + \gamma_t(q_t^T + q_t^H - q_t) 
+ \nu_t^H q_t^H + \nu_t^T q_t^T + \nu_t^f f_t \right\} + \mu \left( \sum_{t=1}^{2} f_t - \sum_{t=1}^{2} q_t^H \right).$$

The first order conditions of P.1 are

$$q_t^T : \quad c'(q_t^T + \alpha f_t) = \gamma_t + \nu_t^T \quad \text{t = 1, 2}$$

$$q_t^H : \quad \mu = \gamma_t + \nu_t^H \quad \text{t = 1, 2}$$

$$f_t : \quad \alpha c'(q_t^T + \alpha f_t) = \mu + \nu_t^f \quad \text{t = 1, 2}$$

$$\gamma_t \geq 0, \quad q_t^T + q_t^H - q_t \geq 0, \quad \gamma_t(q_t^T + q_t^H - q_t) = 0, \quad \text{t = 1, 2}$$

$$\mu \geq 0, \quad f_1 + f_2 - q_1^H - q_2^H \geq 0, \quad \mu(f_1 + f_2 - q_1^H - q_2^H) = 0$$

$$\nu_t^H \geq 0, \quad q_t^H \geq 0, \quad \nu_t^H q_t^H = 0 \quad \text{t = 1, 2}$$

$$\nu_t^T \geq 0, \quad q_t^T \geq 0, \quad \nu_t^T q_t^T = 0 \quad \text{t = 1, 2}$$

$$\nu_t^f \geq 0, \quad f_t \geq 0, \quad \nu_t^f f_t = 0 \quad \text{t = 1, 2}$$

**Proof of Lemma 1**

1. Assume that \(q_t > 0\) and suppose that (3.1) is satisfied as a strict inequality implying that \(\gamma_t = 0\).

Since \(q_t > 0\), then:
- either \( q_t^T > 0 \) so that \( \nu_t^T = 0 \),
- or \( q_t^T = 0 \), but then \( q_t^H > 0 \) so that \( \nu_t^H = 0 \).

1.a. Consider the first case \( q_t^T > 0 \) and \( \nu_t^T = 0 \). Then (A1) results in:
\[
c'(q_t^T + \alpha f_t) = 0
\]
Because \( q_t^T > 0 \), we have \( c' > 0 \), hence a contradiction and we are left with the second case.

1.b. Consider now the second case \( q_t^T = 0, q_t^H > 0 \) and \( \nu_t^H = 0 \). Then, by (A2):
\[
\mu = 0
\]
Hence (A3) results in:
\[
\alpha c'(\alpha f_t) = \nu_t^f.
\]
Assume that \( f_t > 0 \), that is \( \nu_t^f = 0 \). The above equation is now:
\[
\alpha c'(\alpha f_t) = 0
\]
with \( c' > 0 \) because \( f_t > 0 \) and \( \alpha > 0 \); hence, a contradiction. Thus part b of lemma 1 is proved.

However, because \( q_t^H > 0 \) and \( f_t = 0 \), we must have:
- either \( f_{t-1} > 0 \) if \( t = 2 \)
- or \( f_{t+1} > 0 \) if \( t = 1 \)

in order to stay within the same day \( \tau \) so that \( \gamma_t = 0 \) whatever the case.

Hence:
- \( f_{t-1} > 0 \), then \( \nu_t^f = 0 \) so that (A3) results in:
\[
\alpha c'(q_{t-1}^T + \alpha f_{t-1}) = 0,
\]
a contradiction since \( f_{t-1} > 0 \) implies \( c' > 0 \).
- if $f_{t+1} > 0$, then $\nu_{t+1}^f = 0$ so that (A3) results in:

$$\alpha c'(q_{t+1} + \alpha f_{t+1}) = 0,$$

again a contradiction.

2. Let us assume now that (3.2) is satisfied as a strict inequality so that $\mu = 0$, hence, by (A2), (A4) and (A6), both $\gamma_t = 0$ and $\nu_t^H = 0$. Since $\gamma_t = 0$ the proof runs along the same lines than the proof of the above point 1.

**Proof of Lemma 2**

Suppose that $q_t \geq q'_v > 0$ and $q_t^H > 0$. By Lemma 1.b we must have $f'_v = 0$, hence by lemma 1.a and (3.2) as an equality by Lemma 1.a, $f_t = q_t^H + q_t^T$ implying that $f_t > 0$ and, by Lemma 1.b again, that $q_t^H = 0$. Thus $f_t = q_t^H$ and (3.1) as an equality by Lemma 1.a: $q_t^T = q_v - q_t^H$ and $q_t^T = q_t$. From $q_t \geq q'_v$, we trivially get $q_t + \alpha f_t > q_v - q_t^H$.

i) $q_t^H > 0 \Rightarrow \nu_t^H = 0$ so that, by (A2), $\mu = \gamma_v$

ii) Since $q_t^T = q_t$, then $\alpha c'(q_t^T + \alpha f_t) = \alpha c'(q_t + \alpha f_t)$. Futhermore:

$$f_t = q_t^H > 0 \Rightarrow \nu_t^f = 0,$$ so that, by (A3), $\alpha c'(q_t + \alpha f_t) = \mu$.

iii) From $f'_v = 0$ and $q_t^T = q_v - q_t^H$, we get, by (A1):

$$\gamma_v = c'(q_v - q_t^H) - \nu_t^T$$

Summing up, we obtain:

$$\alpha c'(q_t + \alpha f_t) = \mu = \gamma_v = c'(q_v - q_t^H) - \nu_t^T$$

hence a contradiction since $\alpha > 1$, $\nu_t^T \geq 0$ and $c'(q_t + \alpha f_t) \geq c'(q_v + q_t^H)$ this last inequality resulting from the convexity of $c$ and $q_t + \alpha f_t > q_v - q_t^H$.

**Proof of Lemma 3**

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**Necessity:** Assume $f_t = f_{t'} = 0$. We can deduce from (3.2) is an equality, that $q_{t}^{H} = q_{t'}^{H} = 0$ and, consequently, that $q_{t}^{T} = q_{t'} > 0$ and $q_{t'}^{T} = q_{t'} > 0$.

For period $t$, we can write from (A1) $c'(q_{t}) = \gamma_{t}$ so that $\mu = c'(q_{t}) + \nu_{t}^{H}$ by (A2). At date $t'$, from (A3) we have $\alpha c'(q_{t'}) = \mu + \nu_{t'}^{H}$. Consequently, $c'(q_{t}) + \nu_{t}^{H} = \mu = \alpha c'(q_{t'}) - \nu_{t'}^{f} \Rightarrow c'(q_{t}) \leq \alpha c'(q_{t'})$.

**Sufficiency:** We already know by Lemma 2 that $f_{t} > 0$ does not satisfy the first order conditions when $q_{t} \geq q_{t'}$. Let us prove that it is also true for $f_{t'} > 0$ as long as $c'(q_{t}) \leq \alpha c'(q_{t'})$.

Suppose that $f_{t} = 0$ and $f_{t'} > 0$. Then by (3.2) $f_{t'} = q_{t}^{H} + q_{t'}^{H}$, and by Lemma 1.b, $q_{t}^{H} = 0$. Thus $q_{t}^{T} = q_{t'}, q_{t}^{H} = f_{t'}$ and $q_{t'}^{T} = q_{t} - q_{t}^{H}$.

Then 1.b:

$$
\begin{align*}
    f_{t'} > 0 & \implies \nu_{t'}^{f} = 0 \\
    q_{t'}^{T} = q_{t'} & \implies \alpha c'(q_{t'}) + \alpha f_{t'} = \mu \quad \text{by (A3)},
\end{align*}
$$

$$
\begin{align*}
    q_{t}^{H} > 0 & \implies \nu_{t}^{H} = 0 \implies \mu = \gamma_{t} \quad \text{by (A2)},
    q_{t}^{T} = q_{t} - q_{t}^{H} \implies \gamma_{t} = c'(q_{t} - q_{t}^{H}) - \nu_{t}^{f} \quad \text{by (A1)},
\end{align*}
$$

from which we obtain

$$
\alpha c'(q_{t'}) + \alpha f_{t'} = \mu = \gamma_{t} = c'(q_{t} - q_{t}^{H}) - \nu_{t}^{f} \leq c'(q_{t} - q_{t}^{H}).
$$

Since $c''(.) > 0$, then $c'(q_{t'}) < c'(q_{t'}) + \alpha f_{t'}$ and $c'(q_{t} - q_{t}^{H}) < c'(q_{t})$. Thus we get:

$$
\alpha c'(q_{t'}) < c'(q_{t}).
$$
Proof that for \((q_1, q_2)\) solution of \(P.2\) we must have \(q_1 < q_2\). Let \((q_1, q_2)\) be a solution of the optimal dispatching problem \(P.2\). Then by the Inada condition \(u'_t(0) = +\infty, t = 1, 2\), we must have \(q_t > 0, t = 1, 2\). Since any optimal policy must be efficient, then by Lemma 1.a:

\[q_t^T + q_t^H = q_t > 0, \quad t = 1, 2\]

so that:

\[\nu_t^H \nu_t^T = 0,\]

because either \(q_t^H > 0\), or \(q_t^T > 0\) or both, \(t = 1, 2\). For the same reason we must have:

\[f_1 + f_2 = q_1^H + q_2^H.\]

Now assume that \(q_1 \geq q_2\). Then by efficiency and Lemma 2, it must the case that:

\[q_2^H = 0\]

Then by Lemma 1.b, either \(q_1^H > 0\), in which case \(f_1 = 0\), or \(q_1^H = 0\). Hence we must have:

\[
\begin{align*}
q_1 &= q_1^T + q_1^H = q_1^T + f_2 > 0 \\
q_2 &= q_2^T - \alpha f_2 = q_2^T - \alpha q_1^H > 0
\end{align*}
\]

where \(f_2 \geq 0\).

Note that, whatever the value of \(f_2\), \(q_1 \geq q_2\) and the assumptions about \(u_t, t = 1, 2\), imply that:

\[u'_2(q_2) \geq u'_2(q_1) > u'_1(q_1) \quad \text{(A9)}\]

If \(f_2 = 0\), then \(q_t^T = q_t > 0, \quad t = 1, 2\), so that \(q_1 \geq q_2\) and \(c'' > 0\) together imply:

\[c'(q_1) \geq c'(q_2).\]

Since then \(\nu_t^T = 0, \quad t = 1, 2\), taking (A9) into account, we get by (4.1):

\[u'_2(q_2) > u'_1(q_1) = c'(q_1) \geq c'(q_2) = u'_2(q_2),\]

a contradiction.

Next, if \(f_2 > 0\), then \(q_2^T = q_2 + \alpha f_2 > 0\) so that \(\nu_t^T = 0\) and because \(q_2^H = 0, \nu_t^H > 0\) is not necessarily excluded. Hence (4.1)-(4.2) for \(t = 2\) result in:

\[u'_2(q_2) = c'(q_2^T) = \mu - \nu_t^H \leq \mu.\]
Since $f_2 = q_1^H > 0$, then $\nu_1^H = 0$ so that, for $t = 1$, (4.2) results in:

$$u_1'(q_1) = \mu.$$ 

Last taking (A9) into account, we obtain:

$$u_2'(q_2) \leq \mu = u_1'(q_1) < u_2'(q_2),$$

again a contradiction.