Thomas Robert Malthus and His 1798 Theory of Oscillations

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Abstract
This paper formalizes Malthus’s 1798 theory of population using a simple mathematical model and proposes a view of Malthus’s theory of oscillations that is different from both Eltis (2000 [1984]) and Waterman (1987). Furthermore, we apply the account of Malthus’s time lag between the rise in nominal wage rate and the rise in nominal food price to the issue of oscillations. Through this analysis, we can clearly explain how oscillations are influenced by England’s poor laws and we emphasise his statement that it is most important to increase the investment in agriculture, but not in manufacturing.

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1. Introduction
The theory of population presented by Thomas Robert Malthus in the first edition of Principle of Population (1798) has had great influence on the progress of economic and political thoughts. In relation to the issue of population, Malthus provided an account of economic fluctuations of the kind he himself called ‘oscillation’,1 for which various interpretations and mathematical reconstructions have ever been offered by many modern authors. Representative studies may be seen in research by Eltis (2000 [1984]) and Waterman (1987).

Eltis (2000 [1984]: 117, 142) described Malthusian oscillations as the continuing fluctuations in real wages and rates of population growth which are represented by a ‘cobweb’ diagram, arguing ‘because adjustment of population to the wage occurred only after a lengthy time-lag, wages and population tended to fluctuate around these equilibrium value’. On the other hand, Waterman (1987: 265, 269) argued that Malthus’s account of oscillations ‘is not a dynamic cobweb’ about a ‘stationary equilibrium’ or ‘a steady state equilibrium’ of the real wage and rate of population growth, ‘but is a concertina-like edging towards stationary equilibrium’, as described by ‘the zig-zag path of real wages’ between their maximum and minimum. Hollander (1997: 46) objected to Waterman’s account of Malthusian oscillations superimposed onto ‘a land scarcity-based model’ on the main grounds that Malthus’s ‘oscillatory
account makes no mention of increasing land scarcity’. Moreover, these reconstructions of Malthusian oscillations do not explain the difference between nominal and real wages, which Malthus considered as the main cause of oscillations in the first place.

In this paper, we formalize the Malthusian population theory found in the First Essay using a simple mathematical model and propose a view of Malthus’s theory of oscillations that is different from both Eltis and Waterman. The main feature in the paper is to develop a simultaneous differential equations model of Malthus’s population dynamics in order to explore how the difference between nominal and real wages causes oscillations. There are two differential equations for price and population dynamics. The former is the function of the difference between demand for food and its supply and the latter is the function of the real wage represented by the ratio between the nominal wage and price of food. Furthermore, regarding the issue of oscillations, we apply Malthus’s argument that a change in the nominal wage rate always preceded the change in price of food. Through this analysis, we can clearly explain how the oscillations were influenced by England’s poor laws, which Malthus considered as one of the minor causes, and we emphasise his statement presented throughout the First Essay that it is most important to increase the investment in agriculture, but not in manufacturing.

This paper is organised as follows. In section 2, we provide a simple mathematical model to analyse Malthusian oscillations. In section 3, we offer a view of Malthus’s theory of oscillations that is different from both Eltis and Waterman. In section 4, we examine Malthus’s idea of encouraging investment in agriculture using our model. Our conclusion is presented in section 5.

2. The Model
In order to analyse the Malthusian theory of oscillations using a simple mathematical model, let us start with the fundamental assumptions.

First, it is assumed that two main classes exist in the society: ‘a class of proprietors’ who possess land and/or capital and ‘a class of labourers’ who possess ‘no other property than their labour’ (Malthus 1966 [1798]: 345). The object of our model is the latter, that is, the working population, because it ‘is the most numerous class in every nation’ (304).

Second, we suppose that society produces only two kinds of outputs: food (or provisions) as the most important subsistence and, additionally, manufactured luxury items ‘that only tend to gratify the vanity of a few rich people’ (332). In our model, the former is consumed solely by workers and the latter solely by proprietors (to avoid complexity), though food may be consumed by all members of the society. Malthus
also referred to manufactured produce such as ‘the clothing and lodging’ necessary for life, yet he ignored it and assumed manufactured produce to be ‘[t]he fine silks and cottons, the laces, and other ornamental luxuries’ (328-9). Therefore, it follows that no manufactures are included in wage-goods. Moreover, in his explanation, manufactured luxuries are purchased through trade and consumed by foreigners. However, they are omitted in the system because Malthus himself dealt exclusively with the case of a closed economy in his account of oscillations.

From these assumptions, we solely take into consideration the production function for food. Let us now build a mathematical model to analyse the Malthusian theory of oscillations in the First Essay.

Supply
First, we begin with the following assumptions for the supply of food:
(i) In the agricultural industry, labour is the only factor, and thus both fixed capital and raw material are not required.
(ii) The technology in the production of food is exogenously given and equal to the technology at time 0, \( A_0 \in \mathbb{R}^+ \).
(iii) When agricultural population increases, the supply of food, \( S \in \mathbb{R}_+ \), increases in proportion to it. However, when only manufacturing population increases, the supply of food remains the same as the quantity produced by agricultural population at time 0.
(iv) The ratio of the agricultural population to the total population, \( e \in (0,1) \), is constant, so that ‘the exchange of profession’ (309) is out of our consideration.

Thus

\[
S = \begin{cases} 
A_0 e N & \text{if there is an increase in agricultural population,} \\
A_0 e N_0 & \text{if there is not,}
\end{cases}
\]

(1)

where \( N \in \mathbb{R}_+ \) is the total population including the agricultural and manufacturing population and \( N_0 \in \mathbb{R}_{++} \) is the total population at time 0, which is exogenously given.\(^2\)

In our model, we assume the technical production function for food with constant returns to scale, even though some modern writers, Stigler (1952: 190), Waterman (1987: 260; 1991: 264) and Pingle (2003: 6), have opted for deriving production functions with diminishing returns from Malthus’s illustration: ‘The human species would increase in the numbers 1, 2, 4, 8, 16, 32, 64, 128, 256, and subsistence as 1, 2, 3, 4, 5, 6, 7, 8, 9’ (Malthus 1966 [1798]: 25). However, ‘the numbers’ of the ‘human species’, here means the total population, \( N \), and not the agricultural population, \( eN \).
Therefore, we cannot derive an agricultural production function with diminishing returns from Malthus’s ‘geometrical’ and ‘arithmetical’ ratios (18). In fact, in the First Essay, Malthus made no clear statement regarding diminishing returns in agriculture. It was only in the Second Essay on Population that Malthus referred to diminishing returns in agriculture:

Man is necessarily confined in room. When acre has been added to acre till all the fertile land is occupied, the yearly increase of food must depend upon the amelioration of the land already in possession. This is a stream which, from the nature of all soils, instead of increasing, must be gradually diminishing. (13)

Hollander (1997: 16) deals with the long debate regarding whether Malthus’s ‘geometrical’ and ‘arithmetical’ ratios reflected the law of diminishing returns in agriculture, arguing that ‘[t]here is little justification for reading into the ratios a tight functional dependency of the type envisaged by Stigler and Waterman’. As Hollander points out, because Malthus made no mention of increasing land scarcity in his account of oscillations, the diminishing-returns model should not be included.

Demand

In Malthus’s system, a worker was paid in money and not in kind, as he says: ‘The labourer who earns eighteen pence a day’; ‘the eighteen pence a day which men earn’; and ‘the lower classes… received only eighteen pence a day’ (Malthus 1966 [1798]: 67, 75, 78). On the other hand, Malthus said: ‘It very rarely happened that the nominal price of labour universally falls; but we well know that it frequently remains the same, while the nominal price of provisions has been gradually increasing’ (34). Thus, we can assume that $w \in \mathbb{R}_{++}$ is the nominal wage per capita which is exogenously given. As we will refer to later, in Malthus’s own hypothetical illustration, the wage rate is exogenously raised as the result of increased investment by ‘a nation’ (307-8).

The aggregate demand for food is defined so that labour wages must be expended in the purchase of food:

$$pD = wN,$$

(2)

where $D \in \mathbb{R}_+$ is the aggregate demand for food, and $p \in \mathbb{R}_+$ is the price of food.

Dynamics of Price

Malthus states that ‘if the yearly stock of provisions in the country was not increasing’ while the demand for them was simultaneously increasing, then ‘the price of provisions
must necessarily rise’ (307-8). Here he obviously considers that if the demand for food exceeds its supply, the price will rise. Thus, we have to consider the following dynamic equation, which is assumed to be continuously differentiable:

\[ p = f(D - S), \quad f'(\cdot) > 0, \quad f(0) = 0, \quad (3) \]

where a dot over a variable denotes a derivative with respect to time \( t \): \( \dot{p} = dp/dt \).

**Dynamics of Population**

Many scholars often consider that, in Malthusian population theory, population changes depend on whether the real wage rate (which Malthus calls ‘real price of labour’) is above or below the fixed minimum level of subsistence. That is, population grows when the real wage rate is above the level of subsistence and shrinks when the former is below this level and is constant when both are equal. Moreover, Pasinetti (1960), Casarosa (1982), Eltis (2000 [1984]), Dome (1992), Pingle (2003), whose mathematical models included not only Malthus but also Ricardo, frequently represented real wages as the *quantity of food or corn* in order to simplify explanation. They used a population dynamic function which attributed changes in population to real wages in terms of food: the rate of change in population is a function of the real wage rate in terms of food minus the minimum wage rate.

However, we have to note that in Malthus’s system, a worker was paid in *money* and that Malthus considered the time lag between the exogenous change in the nominal wage and endogenous change in the price of food: ‘a rise in the [nominal] price of labour, had preceded the rise in provision’ (Malthus 1966 [1798]: 310). Therefore, if the price of food rises (falls) at any given nominal wage and thereby the real wage is above (below) the fixed minimum level of subsistence, then population increases (decreases); whereas if the nominal wage rate is exogenously raised (lowered) at any given price of food and thereby the real wage is above (below) the fixed minimum level of subsistence, then population increases (decreases).

Thus, we have to consider the following dynamic population equation, which is assumed to be continuously differentiable:

\[ \dot{N} = g \left( \frac{w}{p} - \omega_{\text{min}} \right), \quad g'(\cdot) > 0, \quad g(0) = 0, \quad (4) \]

where \( \dot{N} = dN/dt \) and \( \omega_{\text{min}} \in \mathbb{R}_{++} \) is the minimum level of subsistence predetermined by biological or psychological factors; thus, it is constant through time. The dynamic population equation given in (4) is similar to the one used by Watarai (1988) though his original equation is not continuous but rather discrete.

If the price of food at time 0, \( p_0 \in \mathbb{R}_{++} \), is given, from (1)-(4), six variables \( (S, N, \ldots) \),
$D$, $p$, $\dot{p}$, and $\dot{N}$) are determined. Our model assumes full employment of labour. Watarai (1997: 9) points out that Malthus, in his *First Essay*, tacitly assumes full employment of labour.

3. **Malthusian Theory of Oscillations**

It is well known that Malthus (1966 [1798]: 11) made ‘two postulata’: ‘First, that food is necessary to the existence of man. Secondly, that the passion between the sexes is necessary, and will remain nearly in its present state’. Based on these postulata, he provided the following illustration of oscillations:

We will suppose the means of subsistence in any country just equal to the easy support of its inhabitants. The constant effort towards population, which is found to act even in the most vicious societies, *increases the number of people before the means of subsistence are increased*. The food therefore which before supported seven millions, must now be divided among seven millions and a half or eight millions. The poor consequently must live much worse, and many of them be reduced to severe distress. The number of labourers also being above the proportion of the work in the market, the price of labour must tend toward a decrease; while the price of provisions would at the same time tend to rise. The labourer therefore must work harder to earn the same as he did before. During this season of distress, the discouragements to marriage, and the difficulty of rearing a family are so great, that population is at a stand. In the mean time the cheapness of labour, the plenty of labourers, and the necessity of an increased industry amongst them, encourage cultivators to employ more labour upon their land; to turn up fresh soil, and to manure and improve more completely what is already in tillage; till ultimately the means of subsistence become in the same proportion to the population as at the period from which we set out. The situation of the labourer being then again tolerably comfortable, the restraints to population are in some degree loosened; and the same retrograde and progressive movements with respect to happiness are repeated. (29-31; italics added)

In this illustration, Malthus supposed that population may increase before food increases. We, in the first place, consider a case with no increase in the agricultural population, and we let the economy start from the condition $\omega_{\text{min}}/e = A_0$. To ensure that the quantity of food produced by the agricultural population is sufficient to maintain the entire population, including both the manufacturing and agricultural population,
\( \omega_{\text{min}} < A_0 \) must hold. An economy starting from the case \( \omega_{\text{min}}/e = A_0 \) implies starting from the condition that the quantity of food produced by agricultural population can, during the initial time period, supply the minimum rate of food for maintenance of the total population. This reflects Malthus’s initial supposition that ‘the means of subsistence [are] just equal to the easy support of its inhabitants’.

Substituting (1) and (2) into (3) and considering \( \omega_{\text{min}} = eA_0 \), we obtain

\[
\dot{p} = f \left( \frac{w}{p} N - \omega_{\text{min}} N_0 \right).
\]

(5)

From the two differential equations, (4) and (5), we can derive a solution \((N^*, p^*)\) that brings the dynamic mechanism to a standstill.

However, in order to ensure the existence of interior solutions for this system, in addition to \( N_0 > 0, \omega_{\text{min}} > 0, w > 0 \), a somewhat strong condition, \( \omega_{\text{min}}/\omega_{\text{min}}N_0 \), is required. The function that satisfies \( \dot{p} = 0 \) is linear and described as \( p = wN/\omega_{\text{min}}N_0 \). On the contrary, for population to be stationary, i.e. \( \dot{N} = 0 \), \( w/p \) must be equal to \( \omega_{\text{min}} \). Thus, \( p = w/\omega_{\text{min}} \). As long as \( N_0 > 0 \), \( \omega_{\text{min}} > 0 \) and \( w \in (0, \infty) \) are constant, \( w/\omega_{\text{min}} > 0 \) and \( w/\omega_{\text{min}}N_0 \in (0, \infty) \) are satisfied; therefore, there must exist a stable solution which is necessarily unique.\(^7\)

From (4) and (5), we can find:

\[
\dot{N} \geq 0 \text{ if } \frac{w}{\omega_{\text{min}}} \geq 0, \text{ and } \dot{p} \geq 0 \text{ if } \frac{w}{\omega_{\text{min}}N_0} \geq 0.
\]

Thus, we can describe the phase diagram for the dynamics of \( N \) and \( p \) in Figure 1. In this figure, the \( \dot{N} = 0 \) and \( \dot{p} = 0 \) loci divide the space into four regions. The arrows show the directions of motion in each region. For example, starting from point \( B \), the system converges to the steady state, point \( A \), in an oscillating manner. Returning to point \( A \) represents that ‘ultimately the means of subsistence’ is ‘in the same proportion to the population as at the period from which we set out’, that is, ‘the secular constancy of product per capita’, which Hollander (1998: 43) considers as ‘the striking feature of the account’.

Malthus (1966 [1798]: 31, 34) also states that ‘[t]his sort of oscillation will not be remarked by superficial observers’, and he adds that ‘particularly, the difference between the nominal and real price of labour... has perhaps more than any other [causes], contributed to conceal this oscillation from common view’.

Now we apply the illustration, by Malthus, of a time lag between the rise in the nominal wage rate and rise in the price of food to the issue of oscillations, to make clearer that the main cause of oscillations is the difference between the nominal and real wages.
‘Now supposing a nation, for a course of years, was to add what it saved from its yearly revenue, to its manufacturing capital solely, and not to its capital employed upon land’, that is, when investment is made only in the manufacturing industry, Malthus writes:

There would… be a demand for labour... This demand would of course raise the price of labour; but if the yearly stock of provisions in the country was not increasing, this rise would soon turn out to be merely nominal, as the price of provisions must necessarily rise with it. (307-8)

In this passage, we can find the following mechanism: starting from the situation that the real wage rate equals the subsistence level when the nominal wage in manufacture is raised under a given price of food, population reacts to this change and increases. Because the manufacturing population increases while the agricultural population remains the same, the food demand by the total population increases, and thus the demand for food remains above supply rate. Therefore, the price of food rises and real wages that had also risen in turn falls. However, since the constant supply of food is insufficient for maintenance of the total population, the price of food rises more than the nominal wage, and thus the real wage rate remains below the subsistence level. The population then decreases. This sequence repeats itself until the system converges to the
previous situation. To what degree a population oscillates, namely, how a population reproduces itself depends on the degree to which population reacts to the deviation between the real wage rate and subsistence level; the degree to which the price of food reacts to the deviation between demand and supply and the magnitude of the change in the nominal wage rate being exogenously raised.

We can describe this oscillatory process with our model in Figure 2. If the nominal wage rate is raised by increasing investment solely to manufacture, from \( w \) to \( w^*(>w) \), then the system converges from an initial steady state, point \( A \), to a new one, point \( C \), in an oscillating manner.

On the other hand, Malthus criticized England’s poor laws, reflecting upon the ‘true cause why the immense sum collected in England for the poor does not better their condition’ (71). ‘The poor laws of England’, even if ‘by a subscription of the rich, the eighteen pence a day which men earn now, was made up [of] five shillings’, ‘tend to depress the general condition of the poor’, because they would ‘increase population wi-
Throughout increasing the food for its support’ (75-84). That is, England’s poor laws had the same effect on population as a rise in the nominal wage of manufacturing worker in our present model. Malthus did consider ‘poor laws’ as one of the causes of oscillations (34).

4. Further Considerations

Next, we suppose that investment is made only in agriculture, and thus the nominal wage is raised, although Malthus himself does not provide such a consideration. In a similar manner, let the economy start from \( \omega_{\text{min}} = eA_0 \), then (3) can be rewritten as

\[
\dot{p} = f \left( \frac{w}{p} N - \omega_{\text{min}} N \right). \tag{6}
\]

From this equation, we can find:

\[
\dot{N} \geq 0 \text{ if } \frac{w}{\omega_{\text{min}}} \geq 0, \text{ and } \dot{p} \geq 0 \text{ if } \frac{w}{\omega_{\text{min}}} \geq 0.
\]

Thus, by (4) and (6), we can describe the phase diagram about the dynamics of \( N \) and \( p \), as in Figure 3.

Let the economy start from the initial point \( A \) where the initial population is given by \( N_0 \). Figure 4 shows the case of rise in nominal wage rates, from \( w \) to \( w^* (> w) \), by increasing investment solely in agriculture. Suppose the system moves from the initial steady state, point \( A \), to a new one, point \( D \). By raising \( w \) under a given price of food, the real wage is above the minimum rate and population increases as shown by (4).
reby, the demand for food temporarily exceeds supply and the price rises. However, since in this case, increase in the supply of food follows, unlike in the case where investment is made only in manufacturing, it is possible to sustain more people than previously possible. Therefore, the economy grows as population increases. That is, such a creation of demand, namely, a rise in the nominal wage by investing in agriculture is effective, but investing in manufacture is not.

Malthus proposed ‘encouragements… to agriculture above manufactures’ as one of the remedies to alleviate the distress of the working classes, although these remedies merely represented ‘a palliative’ (95-6). Our proposed model can explain his statement:

An increase in the price of provisions would arise, either from an increase of population faster than the means of subsistence; or from a different distribution of the money of the society. The food of a country that has been long occupied, if it be increasing, increases slowly and regularly, and cannot be made to answer any sudden demands; but variations in the distribution of the money of a society are not infrequently occurring, and are undoubtedly among the causes that occasion the continual variations which we observe in the price of provisions. (82-3)
After all, in Malthus’s principle of population, it is most important to increase the investment in agriculture. Malthus justly stresses: ‘Increase the demand for agricultural labour by promoting cultivation, and with it consequently increase the produce of the country, and ameliorate the condition of the labourer, and no apprehensions whatever need be entertained of the proportional increase of population’ (133). He also states:

In that [nation] which had applied itself chiefly to agriculture, the poor [laboring classes] would live in great plenty, and population would rapidly increase. In that which had applied itself chiefly to commerce, the poor would be comparatively but little benefited, and consequently population would increase slowly. (326)

5. **Conclusion**

We analysed the Malthusian theory of oscillations using a simple mathematical model. In particular, our model clarifies how the difference between nominal and real wages causes oscillations by setting two differential equations for population and the price of food and then by describing the phase diagram, as shown in Figure 1. Furthermore, we applied the account of Malthus’s time lag between the rise in the nominal wage rate and the rise in the price of food to the issue of oscillations, so that it will be clearer that the main cause of oscillations is the difference between nominal and real wages. In the case where there is no increase in the agricultural population, we suppose that the nominal wage rate is exogenously raised as a result of the investment by ‘a nation’, under the given price. First since the real wage rate rises, population grows, increasing the demand for food while its supply remains the same as before. Then food prices rise, forcing down the real wage rate. Population oscillates until the system converges to a new equilibrium, as shown in Figure 2. Through this analysis, we can explain how the oscillations are influenced by the poor laws, which Malthus considered as one of their minor causes. In turn, on the same suppositions in the case where agricultural population does increase, it is found that the expansion of food supply keeps pace with the increase in population, as shown in Figure 4. This emphasizes the importance of the investment in agriculture as advanced by Malthus in the *First Essay*.

Malthus seems to have intuitively understood that the oscillation was caused by the difference between the nominal and real wages within the time lag between the rise of nominal wages and the rise of food prices.
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NOTES

1. The account of oscillations in Malthus’s First Essay is repeated with minor change in wording in all subsequent editions. See Hollander (1998: 43).
2. In order to avoid any confusion and minimize the use of mathematics, we do not divide the total population between the agricultural and manufacturing population.
3. Blaug (1958: 104) has noted, ‘Malthus never emphasized the law of diminishing returns in the Essay on Population’ and that, in the Summary View of the Principle of Population, ‘Malthus states quite clearly why he is unwilling to resort to decreasing returns as the basis of the arithmetic ratio’. As in this paper, which introduce the production function with constant returns, Dome (1992) also analyses Malthus’s growth theory.
4. Hollander (1997: 46) admits that ‘[t]here does exist in the first Essay a land scarcity-based model which is applied to the American case’, but adds that ‘this entails a declining secular corn wage, whereas the oscillatory process is conspicuous for constancy (on an average) of the corn wage’. As will be shown, the long-term constancy of the real wage rate can be ensured in our model. Hollander (1997: 35) also argues: ‘The diminishing-returns model used to interpret the American case is subject to limitations’.
5. Costabile and Rowthorn (1985: 419) writes: ‘[Malthus] argues as follows. Workers are paid in money and not in kind, and the purchasing power of their wages, therefore, depends on the level of prices at the time these wages are spent. The level of prices in turn depends on the relation between supply and demand in the goods market.’
6. The model assumes the constant value of money.
7. A stable solution in the case in which there is no increase in the agricultural population is defined as $(N^*, p^*) = (N_0, w/\omega_{min})$. By expanding (4) and (5) in a Taylor series around $N^*$ and $p^*$, and neglecting the terms of higher order than the first, we obtain
\[
\begin{pmatrix}
N \\
\dot{p}
\end{pmatrix} = \begin{pmatrix}
0 & -\frac{\omega_{\min}}{p^*} g'(0) \\
\omega_{\min} f'(0) & -\frac{\omega_{\min}}{p^*} N^* f'(0)
\end{pmatrix} \begin{pmatrix}
N - N^* \\
p - p^*
\end{pmatrix}.
\]

Let \( \lambda = (\lambda_1, \lambda_2) \) be the roots of the characteristic equation,

\[
\begin{vmatrix}
0 - \lambda & -\frac{\omega_{\min}}{p^*} g'(0) \\
\omega_{\min} f'(0) & -\frac{\omega_{\min}}{p^*} N^* f'(0) - \lambda
\end{vmatrix} = 0,
\]
or

\[
\lambda^2 + 2x\lambda + y^2 = 0,
\]
where \( x \equiv \frac{\omega_{\min} N f'(0)}{2p^*} > 0 \) and \( y \equiv \frac{\omega_{\min}^2 N^* f'(0) g'(0)}{p^*} > 0 \). Hence if \( \lambda_1 > \lambda_2 \),

\[
\lambda_1 = -x + \sqrt{x^2 - y^2},
\]
\[
\lambda_2 = -x - \sqrt{x^2 - y^2}.
\]

Since the real parts of both eigenvalues are negative, the system is stable.

8. Malthus (1966 [1798]: 282) did agree with Smith ‘that what is saved from revenue is always added to stock’ so that in Malthus’s system, saving and investment are considered to be equal.

9. It is possible that a rise in the nominal wage rate in the manufacturing sector causes a labour turnover from agricultural to manufacturing sectors. As was previously written, Malthus himself referred to this as an ‘exchange of professions’, a somewhat ambiguous phrase. Malthus did not include this in his discussion of oscillations, so the topic remains beyond our consideration. Further research could address that topic.

10. It is noted that Malthus did not absolutely oppose encouragements to manufacturing. One reason is that he did not make light of the effect of ‘[i]mprovements in manufacturing machinery’ (Malthus 1966 [1798]: 308). Further research could also include that topic.

References


